Math 10 F&P
Arithmetic Sequences and Series

Name ★ Key ★

ARITHMETIC SEQUENCES & SERIES WORKSHEET

The **general term of an arithmetic sequence** is given by the formula
$a_n = a_1 + (n - 1)d$ where a_1 is the **first term** in the sequence and **d** is the **common difference**.

Finding the sum of a given arithmetic sequence:

1. Identify t_1, n, and d for the sequence.
2. Find t_n using $t_n = t_1 + (n - 1)d$
3. Substitute and evaluate: $S_n = \frac{n(t_1 + t_n)}{2}$

$S_n = \frac{n}{2}(2t_n + (n-1)d)$

1. Write down the stated term and the nth term of the following arithmetic sequences

a. 7, 11, 15, ... (7^{th})

$t_{15} = 7 + (6)(4)$
$= 7 + 24$
$= 31$

c. 18, 11, 4, ... (6^{th})

$t_6 = 18 + (5)(-7)$
$= 18 - 35$
$= -17$

b. -7, -5, -3, ... (23^{rd})

$t_{23} = -7 + (22)(2)$
$= -7 + 44$
$= 37$

d. 3, 3 ½, 4, ... (16^{th})

$t_{16} = 3 + (15)(0.5)$
$= 10.5$

2. Find the sum of the following series. (hint: use the formula for arithmetic sequences first to find n)

a. 5, 9, 13, ..., 101

$t_n = t_1 + (n-1)d$
$101 = 5 + 4n - 4$
$101 = 1 + 4n$
$\frac{100}{4} = n$
$n = 25$

$S_{25} = \frac{25}{2}(5 + 101)$
$= 1325$

c. 83, 80, 77, ..., 5

$t_n = t_1 + (n-1)d$ $\quad d = -3$
$5 = 83 - 3n + 3$
$5 = 86 - 3n$
$\frac{-81}{3} = \frac{-3n}{-3}$
$n = 27$

$S_{27} = \frac{27}{2}(83 + 5)$
$= 1188$

b. -17, -12, -7,..., 33

$33 = -17 + 5n - 5$
$33 = -22 + 5n$
$+22 \quad +22$
$\frac{55}{5} = \frac{5n}{5}$
$n = 11$

$S_{11} = \frac{11}{2}(-17 + 33)$
$= 88$

d. 1, 1 ¼, 1 ½, ... 9 ¾

$9\frac{3}{4} = 1 + 0.25n - 0.25$
$9.75 = 0.75 + 0.25n$
$-0.75 \quad -0.75$
$9 = 0.25n$
$n = 36$

$S_{36} = \frac{36}{2}(1 + 9.75)$
$S_{36} = 193.50$

3. Find the sum of the following series.

a. 4, 11, ... to 16 terms

$S_{16} = \frac{16}{2}(2(4) + (15)(7))$
$= 904$

c. 3, 8 ½, ... to 20 terms

$S_{20} = \frac{20}{2}(2(3) + (19)(5.5))$
$= 1105$

b. 19, 13, ... to 10 terms

$S_{10} = \frac{10}{2}(19(2) + (9)(-6))$

$= -80$

d. -9, -1, ... to 8 terms

$S_8 = \frac{8}{2}(2(-9) + (7)(8))$

$= 152$

4. Fill in the gaps in this arithmetic sequence: -3, -0.5, 2, 4.5, 7, 9.5, 12

$d = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad d = \frac{12+3}{6} = 2.5$

x	y
1	-3
7	12

5. An arithmetic sequence has a 10th term of 17 and a 14th term of 30. Find the common difference.

x	y
10	17
14	30

$d = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-17}{14-10} = \frac{13}{4} = 3\frac{1}{4}$ (3.25)

6. Find the sum of the first 100 odd numbers

$S_{\text{ODD}_{100}} = \frac{100}{2}(1 + 199)$

$= 10{,}000$

7. Find the sum of the positive terms of the arithmetic sequence 85, 78, 71, ...

$S_{13} = \frac{13}{2}(2(85) + (12)(-7))$

$= \frac{13}{2}(170 + (-84))$ $= 559$

$0 = 85 + (n-1)(-7)$

$0 = 85 - 7n + 7$

$0 = 92 - 7n$

$\frac{-92}{-7} = \frac{-7n}{-7}$ $\quad n = 13.14$

≈ 13

8. The second term of an arithmetic sequence is 7. The sum of the first 4 terms of the arithmetic sequence is 12. Find the first term a_1, and the common difference, d, of the sequence.

$t_2 = 7$ $\quad S_4 = 12$ $\quad n = 4$

$-, 7, -, -$

$S_4 = \frac{n}{2}(t_1 + t_n)$

$12 = \frac{4}{2}(a + a + (4-1)d)$

$12 = 2(2a + 3d)$

$12 = 4a + 6d$

a = 1ST term

d = difference

$4(a + d = 7)$ ← equation #1

$4a + 6d = 12$ ← equation #2

$4a + 4d = 28$

$-\ 4a + 6d = 12$

$-2d = 16$

$d = -8$

$a = 15$

9. The first, second, and the nth terms of an arithmetic sequence are 2, 6, and 58 respectively,

a. Find the value of n

x	y
1	2
2	6
n	58

$t_n = 58$

$n = ?$

$t_1 = 2$

$t_n = t_1 + (n-1)d$

$58 = 2 + (n-1)(4)$

$58 = 2 + 4n - 4$

$58 = -2 + 4n$

$+2 \quad +2$

$\frac{60}{4} = \frac{4n}{4}$

$n = 15$

b. For that value of n, find the exact value of the sum of n terms.

$S_{15} = \frac{15}{2}(2 + 58)$

$= 450$

10. The 10th term of an arithmetic sequence is 10 and the sum of the first 10 terms is -35. Find the first term a_1, and the common difference, d, of the sequence.

$-35 = \frac{10}{2}(2a + (9)(d))$

$-35 = 5(2a + 9d)$

$-35 = 10a + 45d$

x	y
10	10

$t_{10} = 10$

$S_{10} = -35$

$10(a + 9d = 10)$

$10a + 45d = -35$

$10a + 90d = 100$

$-\ 10a + 45d = -35$

$45d = 135$

a = 1ST term

d = difference

$d = 3$

$a = -17$

12. How many terms of the arithmetic sequence {1,3,5,7,...} will give a sum of 961?

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$ $961 = \frac{n}{2}(2(1) + 2n - 2)$

$961 = \frac{n}{2}(2n)$ $961 = \frac{2n^2}{2}$

$\sqrt{961} = \sqrt{n^2}$ $n = 31$

13. Jerry deposited $20,000 on an investment that will give $1,750 for every year that his money stays in the account. How much money will he have in his account by the end of year 8?

20,000, 21750...

$t_n = t_1 + (n-1)d$

$= 20,000 + (8-1)(1750)$

$= 20,000 + 12,250$ $= \$32,250$

$t_1 = 20,000$
$d = 1750$
$n = 8$
$t_n = ?$

14. There is a stack of logs in the backyard. There are 15 logs in the 1st layer, 14 in the second, 13 in the third, 12 in the fourth, and so on with the last layer having one log. How many logs are in the stack?

15, 14, 13, 12 ... 1

$n = 15$

$S_n = \frac{n}{2}(t_1 + t_n)$

$= \frac{15}{2}(15+1)$

$= 120$ logs

15. In his piggy bank, Bingo dropped $1.00 on May 1, $1.75 on May 2, $2.50 on May 3 and so on until the last day of May.

x	y
1	1
2	1.75
3	2.50

$n = 19$
$d = 0.75$
$t_1 = 1$
$t_n = ?$

a) How much did he drop in his piggy bank on May 19?

$t_n = t_1 + (n-1)d$

$= 1 + (18)(0.75)$

$= \$14.50$ on May 19

b) What was his total deposit in his piggy bank for the month of May?

May = 31 days.

$n = 31$

$S_{31} = \frac{31}{2}(2(1) + (31-1)(0.75))$

$= \$379.75$

16. There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. If the price per ticket is $2,300, how much will be the total sales for a one-night concert if all seats are taken?

25, 27, 29 ...

$S_n = ?$
$t_1 = 25$
$n = 20$
$d = 2$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$= \frac{20}{2}(2 \cdot 25 + (19)(2))$

$= 880$ seats

$\$ = 880(2300)$

$= \$2,024,000$

17. Sonia has 55 blocks. She decides to stack up all the blocks so that each row has one less block than the row below. She wants to end up with just 1 block on top. How many should she put in the bottom row?

$S_n = 55$
$d = 1$
$t_1 = 1$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$55 = \frac{n}{2}(2(1) + n - 1)$

$55 = \frac{n}{2}(1+n)$

$2(55) = \left(\frac{n}{2} + \frac{n^2}{2}\right)^2$

$110 = n + n^2$

$n^2 + n - 110 = 0$

$n = -11, 10$

must be positive ↓ 10

18. A theater has 32 rows of seats. If there are 26 seats in the 1st row, 30 in the 2nd, 34 in the 3rd, and so on, how many seats are there in all?

$t_1 = 26$
$n = 36$
$d = 4$

$S_{32} = \frac{32}{2}(2 \cdot 26 + (31)(4))$
$= 16(52 + 124)$
$= 16(176)$
$= 2816$ seats

19. A tube well is bored 800 meters deep. The 1st meter costs \$250 and the cost per meter increases by \$50 for every subsequent meter. Find the cost of boring the 750th meter and the total cost incurred for the entire job.

$n = ~~800~~ 750$
$t_1 = \$250$
$d = \$50$

\$250, \$300, \$350, \$400 . . .

$t_{750} = 250 + (750 - 1)(50)$
$= 250 + (749)(50)$
$= \$37{,}700.$

$S_{800} = \frac{n}{2}(2t_1 + (n-1)d)$
$= \frac{800}{2}(2 \cdot 250 + (800-1)(50))$
$= \frac{800}{2}(500 + 39950)$
$= 400(40450)$
$= \$16{,}180{,}000$